

Discharging PO of **DLF**: Revisiting First Attempt

$$\frac{H(\mathbf{F}), \mathbf{E} = \mathbf{F} \vdash P(\mathbf{F})}{H(\mathbf{E}), \mathbf{E} = \mathbf{F} \vdash P(\mathbf{E})} \text{EQ_LR}$$

$$\frac{H(\mathbf{E}), \mathbf{E} = \mathbf{F} \vdash P(\mathbf{E})}{H(\mathbf{F}), \mathbf{E} = \mathbf{F} \vdash P(\mathbf{F})} \text{EQ_RL}$$

$$\begin{array}{l} d \in \mathbb{N} \\ n \in \mathbb{N} \\ n \leq d \\ \vdash \\ n < d \vee n > 0 \\ \equiv \\ d \in \mathbb{N} \\ n \in \mathbb{N} \\ n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

MON

$$\begin{array}{l} n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

OR.L

$$\begin{array}{l} n < d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

$$\begin{array}{l} n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

$$\begin{array}{l} n < d \\ \vdash \\ n < d \end{array}$$

OR.R1

$$\begin{array}{l} n < d \\ \vdash \\ n < d \end{array}$$

HYP

$$\vdash$$

EQ_LR, MON

$$\vdash$$

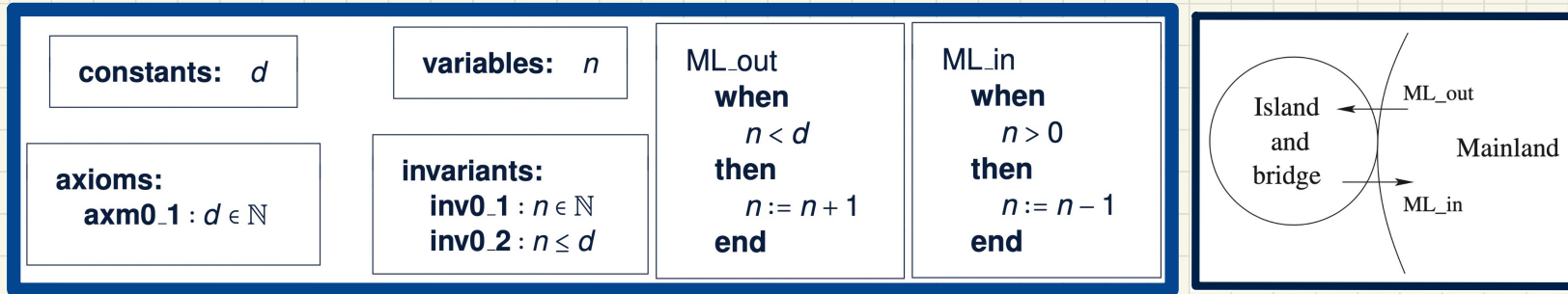
OR.R2

$$\vdash$$

?

what if EQ_RL was used?

Understanding the Failed Proof on **DLF**



Unprovable Sequent: $\vdash d > 0$

Discharging PO of **DLF**: Second Attempt

$$\begin{array}{l} d \in \mathbb{N} \\ n \in \mathbb{N} \\ n \leq d \\ \vdash \\ n < d \vee n > 0 \end{array}$$
 $\equiv$
$$\begin{array}{l} d \in \mathbb{N} \\ n \in \mathbb{N} \\ n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

MON

$$\begin{array}{l} n < d \vee n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

OR_L

$$\begin{array}{l} n < d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

OR_R1

$$\begin{array}{l} n < d \\ \vdash \\ n < d \end{array}$$

HYP

$$\begin{array}{l} n = d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

EQ_LR, MON

$$\begin{array}{l} \vdash \\ d < d \vee d > 0 \end{array}$$

OR_R2

$$\begin{array}{l} \vdash \\ d > 0 \end{array}$$

?

Discharging PO of **DLF**: Second Attempt

$$\frac{}{H, P \vdash P} \text{HYP}$$

$$\frac{H1 \vdash G}{H1, H2 \vdash G} \text{MON}$$

$$\frac{H, P \vdash R \quad H, Q \vdash R}{H, P \vee Q \vdash R} \text{OR_L}$$

$$\frac{H \vdash P}{H \vdash P \vee Q} \text{OR_R1}$$

$$\frac{H \vdash Q}{H \vdash P \vee Q} \text{OR_R2}$$

$$\begin{array}{l} d \in \mathbb{N} \\ d > 0 \\ n \in \mathbb{N} \\ n \leq d \\ \vdash \\ n < d \vee n > 0 \end{array}$$

Summary of the Initial Model: Provably Correct

constants: d

variables: n

axioms:

axm0_1 : $d \in \mathbb{N}$

axm0_2 : $d > 0$

invariants:

inv0_1 : $n \in \mathbb{N}$

inv0_2 : $n \leq d$

init
begin
 $n := 0$
end

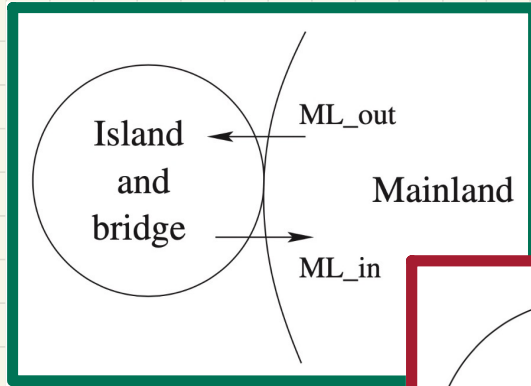
ML_out
when
 $n < d$
then
 $n := n + 1$
end

ML_in
when
 $n > 0$
then
 $n := n - 1$
end

Correctness Criteria:

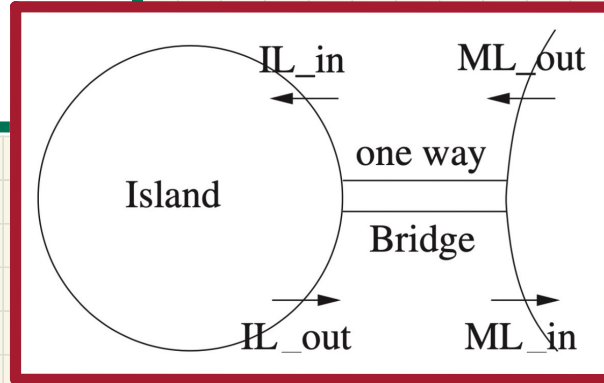
- + Invariant Establishment
- + Invariant Preservation
- + Deadlock Freedom

Bridge Controller: **Abstraction** in the 1st Refinement



m0:

initial, most **abstract**



m1:

second, more **concrete**

REQ1

The system is controlling cars on a bridge connecting the mainland to an island.

REQ3

The bridge is one-way or the other, not both at the same time.

Bridge Controller: State Space of the 1st Refinement

REQ1

The system is controlling cars on a bridge connecting the mainland to an island.

REQ3

The bridge is one-way or the other, not both at the same time.

Dynamic Part of Model

variables: a, b, c

invariants:

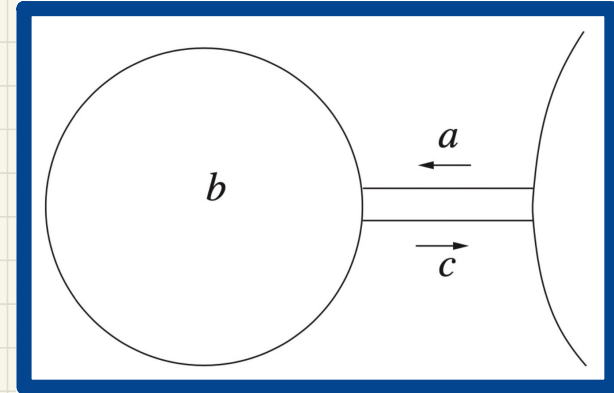
inv1_1 : $a \in \mathbb{N}$

inv1_2 : $b \in \mathbb{N}$

inv1_3 : $c \in \mathbb{N}$

inv1_4 : ??

inv1_5 : ??



Static Part of Model

constants: d

axioms:

axm0_1 : $d \in \mathbb{N}$

axm0_2 : $d > 0$

Exercises

inv1_4: linking abstract & concrete states

inv1_5: bridge is one-way